On the complex solutions to the (3 + 1)-dimensional conformable fractional modified KdV–Zakharov–Kuznetsov equation

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Received 13 July 2019
Revised 17 September 2019
Accepted 8 October 2019
Published 2 January 2020

This paper presents some exponential function solutions of the (3 + 1)-dimensional space-time fractional modified KdV–Zakharov–Kuznetsov (mKdV–ZK). The improved Bernoulli sub-equation function method (IBSEFM) is used in carrying out this computation. Choosing the suitable parameters, the 2D and 3D surfaces of solutions are plotted. The constructed results may be useful in explaining the physical meaning of the studied model.

Keywords: Dynamic systems; (3+1)-dimensional conformable time fractional mKdV–ZK equation; wave solutions.

1. Introduction

Several nonlinear phenomena which arise in the various fields of nonlinear sciences such as fluid dynamics, plasma physics, mathematical biology, finance, elastic media, geochemistry and so on, may be expressed in the form of nonlinear partial differential equations. Fractional differential equations are the general form of differential equations with integer order. Fractional differential equations arise from the most part from the mathematical model of physical phenomena such as viscoelasticity, physics, electrical network, solid state physics, control theory of dynamical systems, chemical physics, optics, engineering applications which are usually modeled with nonlinear differential equations.
There are several definitions of fractional derivative, such as Riemann–Liouville, Caputo, Caputo–Fabrizio, Atangana–Baleanu. Recently, a new definition of local fractional derivative, namely the conformable fractional was developed by Khalil et al. Abdeljawad investigated more on the newly developed conformable fractional derivative. In the literature, there are several studies on analytical and numerical solutions to the various fractional differential equations involving the local conformable derivative.

Several computational techniques for the solutions of different kinds of nonlinear equations have been developed and utilized by different researchers, such as the modified trial equation method (MTEM), first integral method, the exp-function method, the (G’/G)-expansion method, the functional variable method, the undetermined coefficients method, the improved fractional sub-equation method, and many other symbolic techniques that require tedious computations.

In this study, the (3 + 1)-dimensional fractional modified KdV–Zakharov–Kuznetsov (mKdV–ZK) equation is investigated by using the improved Bernoulli sub-equation function method.

The (3 + 1)-dimensional fractional mKdV–ZK equation is given by

\[ \frac{\partial^\gamma u}{\partial t^\gamma} + pu^2 \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial y^2 \partial x} + \frac{\partial^3 u}{\partial z^2 \partial x} = 0, \quad t \geq 0, \quad 0 < \gamma \leq 1, \]  

where \( p \) is a nonzero constant. When \( \gamma = 1 \), Eq. (1) changes to the classical mKdV–ZK equation. The Korteweg–de Vries equation is an important nonlinear problem which is used in modeling weakly nonlinear shallow water waves. The mKdV evolution equation describes the small amplitude ion-acoustic solitons, and this also includes lowest-order nonlinearity and dispersion.

The Zakharov–Kuznetsov (ZK) equation describes the ionic-acoustic waves in magnetized plasmas in two dimensions. The modified ZK (mZK) equation occurs naturally as weakly two-dimensional variation of the mKdV equation. The ZK equation may also be used to describe the nonlinear development of ion-acoustic waves in a magnetized plasma under the restrictions of small wave amplitude, weak dispersion and strong magnetic fields.

**Conformable Fractional Derivative**

**Definition.** Let \( f : [0, \infty) \to \mathbb{R} \), the conformable fractional derivative of order \( \alpha \) is defined as

\[ T_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \]

for all \( t > 0, \alpha \in (0, 1) \).
Theorem. Let $T_\alpha$ be a fractional derivative operator with order $\alpha$ and $\alpha \in (0, 1]$, $f, g$ be $\alpha$-differentiable at point $t > 0$. Then,

(i) $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$, $\forall a, b \in \mathbb{R}$.

(ii) $T_\alpha(t^p) = pt^{p-\alpha}$, $\forall p \in \mathbb{R}$.

(iii) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.

(iv) $T_\alpha(fg) = gT_\alpha(f) - fT_\alpha(g)g^2$.

(v) $T_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$.

(vi) If $f$ is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}(t)$.

2. The Structure of the Method

In this section, we present the general structure of the IBSEFM.

**Step 1.** Suppose that the following fractional differential equation:

$$P(D_t^\alpha u, u_x, u_t, u_{xt}, \ldots) = 0,$$

where $u = u(x, t)$ and $\alpha \in (0, 1]$ is the order of the conformable derivative.

Consider the wave transformation

$$u(x, t) = U(\zeta), \quad \zeta = \sigma x - \mu t^\alpha,$$

where $\sigma, \mu$ are constants which can be determined later. Equation (3) reduces Eq. (2) to the following nonlinear ordinary differential equation (NODE):

$$N(U, U', U'', U''', \ldots) = 0.$$

**Step 2.** Assume that Eq. (4) has trial solution of the form

$$U(\zeta) = \sum_{i=0}^n a_i F_i(\zeta) = \sum_{j=0}^m b_j F_j(\zeta),$$

where $F_i(\zeta)$ and $F_j(\zeta)$ are Bernoulli differential polynomial functions. Substituting Eq. (6) into Eq. (4) gives the following polynomial equation $\Omega(F)$ which depends on $F$:

$$\Omega(F) = \rho_0 F^s + \cdots + \rho_1 F + \rho_0 = 0.$$

We use the homogeneous balance principle to determine the relation between $n, m$ and $M$.

**Step 3.** Equating all the coefficients of $\Omega(F)$ to yields an algebraic equation system

$$\rho_i = 0, \quad i = 0, \ldots, s.$$

Once this system is solved, we can determine the values of $a_0, a_1, \ldots, a_n$ and $b_0, b_1, \ldots, b_m$. 

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Step 4. We have two situations depending on $b$ and $d$ according to solution of Eq. (6).

\[ F(\eta) = \left[ \frac{-d}{b} + \frac{E}{e^{b(M-1)\eta}} \right]^{\frac{1}{1-M}}, \quad b \neq d, \quad (8) \]

\[ F(\eta) = \left[ \frac{(E-1) + (E + 1) \tanh(b(1-M)\eta/2)}{1 - \tanh(b(1-M)\eta/2)} \right]^{\frac{1}{1-M}}, \quad b = d, \quad E \in \mathbb{R}. \quad (9) \]

We get the analytical solutions of Eq. (4) by using complete discrimination system for polynomial of $F(\zeta)$.

3. Application

In this section, we apply the proposed method to the $(3 + 1)$-dimensional time fractional mKdV–ZK equation.

Consider the wave transformation

\[ u(x, y, z, t) = U(\zeta), \quad \zeta = \alpha x + \beta y + \theta z - \frac{\kappa t}{\gamma}, \quad (10) \]

where $\alpha, \beta, \theta$ and $\kappa$ are nonzero constants.

Substituting Eq. (10) into Eq. (1), we have the following NODE:

\[ -\kappa U + \frac{\partial^3 U}{\partial \zeta^3} + (\alpha^3 + \beta^2 \alpha + \theta^2 \alpha)U'' = 0. \quad (11) \]

Using the homogeneous balance principle between $U''$ and $U^3$, we get relationship for $n, m$ and $M$ as follows:

\[ M + m = n + 1. \quad (12) \]

Case 1. Taking $M = 3$ and $m = 1$, gives $n = 3$.

Thus, one may write the trial solution of Eq. (11) as

\[ U = \frac{a_0 + a_1 F + a_2 F^2 + a_3 F^3}{b_0 + b_1 F} = \frac{\Upsilon}{\Psi}, \quad (13) \]

\[ U' = \frac{\Upsilon' \Psi - \Upsilon \Psi'}{\Psi^2}, \quad (14) \]

\[ U'' = \frac{\Upsilon'' \Psi - \Upsilon' \Psi'}{\Psi^2} - \frac{(\Upsilon \Psi')' \Psi^2 - 2 \Upsilon (\Psi')^2 \Psi}{\Psi^4}, \quad (15) \]

where $F' = bF + dF^3, \quad a_3 \neq 0, \quad b_1 \neq 0$.

Putting Eqs. (13)–(15) into Eq. (11), we get a system of algebraic equations. By solving this system of algebraic equations with the help of symbolic software, we get the values of the coefficients involved. Substituting the values of the coefficients into Eq. (13), gives solutions to Eq. (1).
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Case 1(a). For $b \neq d$

$$a_1 = b_1 \sqrt{\frac{3k}{p\alpha}}, \quad a_2 = \frac{2da_0}{B}, \quad b_0 = a_0 \sqrt{\frac{p\alpha}{3k}},$$

$$a_3 = \sqrt{\frac{3k}{p\alpha}} \frac{2db_1}{b}, \quad \theta = \sqrt{\frac{-k - 2b^2\alpha(\alpha^2 + \beta^2)}{2ab^2}},$$

(16)

we have the following solution:

$$u_{1a} = \frac{-3(de^f(x,y,z,t) + bEE)\sqrt{k} \left( \sqrt{-\frac{d}{b}} + e^{-f(x,y,z,t)EE} \sqrt{p\alpha a_0} + \sqrt{3kb_1} \right)}{(de^f(x,y,z,t) - bEE)\sqrt{p\alpha} \left( \sqrt{-\frac{3d}{b}} + 3e^{-f(x,y,z,t)EE} \sqrt{p\alpha a_0} + \sqrt{3kb_1} \right)},$$

(17)

where $f(x, y, z, t) = \frac{\sqrt{2zi} \sqrt{k + 2b^2\alpha(\alpha^2 + \beta^2)}}{\sqrt{\alpha}} + 2b(x\alpha + y\beta - \frac{kt\gamma}{\gamma})$.

Fig. 1. (Color online) The 3D–2D and contour surfaces of Eq. (17) with $\gamma = 0.5$.
Case 1(b). For $b = d$,

$$a_1 = b_1 \sqrt{\frac{3k}{p\alpha}}, \quad a_2 = \frac{2da_0}{B}, \quad b_0 = a_0 \sqrt{\frac{p\alpha}{3k}},$$

$$a_3 = \sqrt{\frac{3k}{p\alpha} \frac{2db_1}{b}}, \quad \theta = \sqrt{-\frac{k - 2b^2\alpha(\alpha^2 + \beta^2)}{2\alpha b^2}},$$

we have the following solution:

$$u_{1b} = -\frac{3 \left( e^{g(x,y,z,t)} + e^{\frac{2dt\gamma}{\gamma}} \right) \left( \sqrt{k(-1 + e^{-h(x,y,z,t)\gamma})}a_0 + \sqrt{3k}b_1 \right)}{\left( e^{g(x,y,z,t)} - e^{\frac{2dt\gamma}{\gamma}} \right) \left( \sqrt{-3 + 3e^{-h(x,y,z,t)\gamma}} \sqrt{p\alpha}a_0 + 3\sqrt{k}b_1 \right)},$$

where $g(x, y, z, t) = 2d(x\alpha + y\beta) + \sqrt{2z} + \sqrt{k + 2d^2\alpha(\alpha^2 + \beta^2)}$, and $h(x, y, z, t) = \frac{\sqrt{2z} + \sqrt{k + 2d^2\alpha(\alpha^2 + \beta^2)}}{\sqrt{\alpha}} + 2d(x\alpha + y\beta - \frac{kt\gamma}{\gamma}).$

Fig. 2. (Color online) The 3D–2D and contour surfaces of Eq. (19) with $\gamma = 0.5$. 
Case 1(c). For $b \neq d$

$$a_1 = -\frac{a_3}{2d} \sqrt{\frac{k}{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}},$$

$$a_2 = -2da_0 \sqrt{\frac{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}{k}}, \quad p = \frac{3kb_0^2}{\alpha a_0^2},$$

$$b_1 = -\frac{a_3b_0}{2da_0} \sqrt{\frac{k}{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}}, \quad B = -\sqrt{\frac{k}{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}},$$

$$u_{1c} = \frac{(e^{2l(x,y,z,t)}EE^2k + 2d^2\alpha(\alpha^2 + \beta^2 + \theta^2))a_0}{(e^{l(x,y,z,t)}EE\sqrt{k} + \sqrt{2d}\sqrt{-\alpha(\alpha^2 + \beta^2 + \theta^2))^2}b_0},$$

where $l(x,y,z,t) = \frac{2\sqrt{2k}(x\alpha + y\beta - \frac{kt\gamma}{Z} + z\theta)}{\sqrt{\alpha(\alpha^2 + \beta^2 + \theta^2)}}$.

Fig. 3. (Color online) The 3D–2D and contour surfaces of Eq. (21) with $\gamma = 0.5$. 

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Case 1(d). For \( b \neq d \)

\[
\begin{align*}
    a_0 &= \frac{ba_2}{2d}, & a_1 &= \frac{ba_2 b_1}{2db_0}, & a_3 &= \frac{a_2 b_1}{b_0}, & p &= \frac{12d^2 kb_0^2}{b^2 \alpha a_2^2}, \\
    \theta &= \frac{\sqrt{-k - 2b^2\alpha(\alpha^2 + \beta^2)}}{\sqrt{2\alpha b}},
\end{align*}
\]

we have the following solution:

\[
\begin{align*}
    u_{1d} &= \frac{b}{2db_0} \left( -1 + \frac{2bEE}{-de^{\psi(x,y,z,t)} + bEE} \right) a_2,
\end{align*}
\]

where \( \psi(x,y,z,t) = \frac{\sqrt{2zi\sqrt{k+2B^2\alpha(\alpha^2 + \beta^2)}}}{\sqrt{\alpha}} + 2B(x\alpha + y\beta - \frac{kt}{\gamma}) \).

Fig. 4. (Color online) The 3D-2D and contour surfaces of Eq. (23) with \( \gamma = 0.5 \).

Case 1(e). For \( b = d \),

\[
\begin{align*}
    a_0 &= \frac{ba_2}{2d}, & a_1 &= \frac{ba_2 b_1}{2db_0}, & a_3 &= \frac{a_2 b_1}{b_0}, & p &= \frac{12d^2 kb_0^2}{b^2 \alpha a_2^2}, & \theta &= \frac{\sqrt{-k - 2b^2\alpha(\alpha^2 + \beta^2)}}{\sqrt{2\alpha b}},
\end{align*}
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\[
\begin{aligned}
  u_{1e} &= -\frac{(e^g(x,y,z,t) + e^{\frac{2dk+\gamma}{\alpha}} EE) a_2}{2(e^g(x,y,z,t) - e^{\frac{2dk+\gamma}{\alpha}} EE) b_0}, \\
  \text{where } g(x, y, z, t) &= 2d(x\alpha + y\beta) + \sqrt{2zi}k\frac{\sqrt{k^2 + 2d^2\alpha(\alpha^2 + \beta^2)}}{\sqrt{\alpha}}.
\end{aligned}
\]  

(25)

Fig. 5. (Color online) The 3D–2D and contour surfaces of Eq. (25) with \(\gamma = 0.5\).

Case 2. Taking \(M = 3\) and \(m = 2\), gives \(n = 2\).

Thus, one may write the trial solution of Eq. (11) as

\[
U = \frac{a_0 + a_1 F + a_2 F^2 + a_3 F^3 + a_4 F^4}{b_0 + b_1 F + b_2 F^2} = \frac{\Upsilon}{\Psi},
\]

(26)

\[
U' = \frac{\Upsilon'\Psi - \Upsilon\Psi'}{\Psi^2},
\]

(27)

\[
U'' = \frac{\Upsilon''\Psi - \Upsilon'\Psi'}{\Psi^2} - \frac{(\Upsilon\Psi')'\Psi^2 - 2\Upsilon(\Psi')^2\Psi}{\Psi^4},
\]

(28)

where \(F' = bF + dF^4\), \(a_4 \neq 0\), \(b_2 \neq 0\).
Putting Eqs. (26)–(28) into Eq. (11), gives a system of algebraic equations. By solving this algebraic system of equations with the help of symbolic software, we obtain different values of the coefficients involved. Substituting the values of the coefficients into Eq. (26) produces the solutions to Eq. (1).

Case 2(a). For $b \neq d$

$$a_0 = \frac{a_1 b_0}{b_1}, \quad a_2 = \frac{a_1 a_4}{a_3} + \frac{a_3 b_0}{b_1}, \quad b_2 = \frac{a_4 b_1}{a_3},$$

$$b = -\sqrt{\frac{k}{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}}, \quad p = \frac{3kb_1^2}{\alpha a^2_1},$$

$$d = -\frac{a_3}{2a_1} \sqrt{\frac{k}{2(-\alpha(\alpha^2 + \beta^2 + \theta^2))}},$$

we have the solution

$$u_{2a} = a_1 \left( -\frac{a_3 + 2e^{l(x,y,z,t)}EEa_1 + 2a_3}{b_1(-a_3 + 2e^{l(x,y,z,t)}EEa_1)} \right),$$

where $l(x,y,z,t) = \frac{2\sqrt{2}\sqrt{(x\alpha + y\beta - \frac{2\gamma^2}{i\sqrt{\alpha(\alpha^2 + \beta^2 + \theta^2)}} + z\theta)}}{i\sqrt{\alpha(\alpha^2 + \beta^2 + \theta^2)}}$.

Fig. 6. (Color online) The 3D–2D and contour surfaces of Eq. (30) with $\gamma = 0.5$. 

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**Case 2(b).** For \(b \neq d\)

\[
\begin{align*}
    a_0 &= \frac{ba_3 b_1}{2db_2}, \\
    a_1 &= \frac{ba_3}{2d}, \\
    a_2 &= \frac{1}{2} a_4 \left( \frac{bb_2 + 2db_0}{db_2} \right), \\
    b_1 &= \frac{a_3 b_2}{a_4}, \\
    \theta &= -\sqrt{-\frac{1}{6} p a_4^2 - 4d^2(\alpha^2 + \beta^2)b_2^2}, \\
    k &= \frac{b^2 p \alpha a_4^2}{12d^2 b_2^2},
\end{align*}
\]

we have the solution

\[
    u_{2b} = \frac{\left( -dB + b^2 e^{b^3 p \gamma \alpha a_4^2 + bd \gamma b_2 (-12d(x \alpha + y \beta)b_2 + \sqrt{6p \alpha^2 + 144d^2(\alpha^2 + \beta^2)b_2^2})} \right)}{2db_2} EE + 2bd a_4
\]

Fig. 7. (Color online) The 3D–2D and contour surfaces of Eq. (32) with \(\gamma = 0.5\).
Case 2(c). For $b \neq d$

$$a_2 = -\frac{\sqrt{3}pa_4 + 8\sqrt{3\alpha d^2(\alpha^2 + \beta^2 + \theta^2)b_0}}{2d\sqrt{2p(-\alpha(\alpha^2 + \beta^2 + \theta^2))}}, \quad a_0 = b_0\sqrt{\frac{3k}{p\alpha}},$$

$$a_1 = -\frac{a_3}{2d}\sqrt{\frac{k}{2(\alpha(\alpha^2 + \beta^2 + \theta^2))}}, \quad b_1 = -\frac{\sqrt{p}a_3}{2d\sqrt{-6(\alpha^2 + \beta^2 + \theta^2)}},$$

$$b_2 = -\frac{\sqrt{p}a_4}{2d\sqrt{-6(\alpha^2 + \beta^2 + \theta^2)}}, \quad b = -\sqrt{-\frac{k}{-2\alpha(\alpha^2 + \beta^2 + \theta^2)}},$$

we have the solution

$$u_{2c} = \frac{(dke^{l(x,y,z,t)}\sqrt{3 + \sqrt{2})i\sqrt{\alpha(\alpha^2 + \beta^2 + \theta^2)} + 2\sqrt{2kd}\alpha(\alpha^2 + \beta^2 + \theta^2))}{(e^{l(x,y,z,t)}\alpha\sqrt{pkd^2(\alpha^2 + \beta^2 + \theta^2)} + \sqrt{2p\alpha d^2\alpha(\alpha^2 + \beta^2 + \theta^2)}},$$

where $l(x, y, z, t) = \frac{2\sqrt{2k}(x\alpha + y\beta + \frac{z\gamma}{\alpha(\alpha^2 + \beta^2 + \theta^2)})}{i\sqrt{\alpha(\alpha^2 + \beta^2 + \theta^2)}}$.

Fig. 8. (Color online) The 3D–2D and contour surfaces of Eq. (34) with $\gamma = 0.5$.  

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**Case 2(d).** For \(b \neq d\)

\[
a_1 = \frac{ba_3}{2d}, \quad a_2 = \frac{4d^2a_0 + b^2a_4}{2db}, \quad b_0 = a_0 \sqrt{\frac{p\alpha}{3k}}, \quad b_1 = \frac{ba_3}{2d} \sqrt{\frac{p\alpha}{3k}},
\]

\[
b_2 = \frac{ba_4}{2d} \sqrt{\frac{p\alpha}{3k}}, \quad \beta = -\frac{\sqrt{-k - 2b^2\alpha(\alpha^2 + \theta^2)}}{\sqrt{2\alpha b}},
\]

we have the solution

\[
u_{2d} = \frac{-d\sqrt{3k} + \sqrt{3k}be^{\vartheta(x,y,z,t)}EE + \sqrt{3k}d}{-d\sqrt{p\alpha} + b\sqrt{p\alpha}e^{\vartheta(x,y,z,t)}EE},
\]

where \(\vartheta(x,y,z,t) = \frac{2bkt^7}{\gamma} - 2b(x\alpha + z\theta) + \frac{\sqrt{2yi\sqrt{k + 2b^2\alpha(\alpha^2 + \theta^2)}}}{\sqrt{\alpha}}.\)

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Fig. 9. (Color online) The 3D–2D and contour surfaces of Eq. (36) with \(\gamma = 0.5\).
Case 2(e). For \( b = d \),

\[
\begin{align*}
\alpha_1 &= \frac{ba_3}{2d}, \quad \alpha_2 = \frac{4d^2a_0 + b^2a_4}{2db}, \quad \alpha_0 = a_0 \sqrt{\frac{p\alpha}{3k}}, \\
\beta_1 &= \frac{ba_3}{2d} \sqrt{\frac{p\alpha}{3k}}, \quad \beta_2 = \frac{ba_4}{2d} \sqrt{\frac{p\alpha}{3k}}, \\
\beta &= -\frac{\sqrt{-k - 2b^2\alpha(\alpha^2 + \theta^2)}}{\sqrt{2}\alpha b}, \quad b = d,
\end{align*}
\]

we have the solution

\[
\begin{align*}
u_{2e} &= \frac{\sqrt{3k}(-1 + e^{\theta(x,y,z,t)} EE + 2)}{\sqrt{p\alpha}(-1 + e^{\theta(x,y,z,t)} EE)},
\end{align*}
\]

where \( \theta(x,y,z,t) = \frac{2dkt}{\gamma} - 2d(x\alpha + z\theta) + \frac{\sqrt{2}yi\sqrt{k + 2d^2\alpha(\alpha^2 + \theta^2)}}{\sqrt{\alpha}}. \)

Fig. 10. (Color online) The 3D–2D and contour surfaces of Eq. (38) with \( \gamma = 0.5 \).
4. Physical Interpretation of the Figures

In order to have clear and good understanding of the physical properties of the constructed complex exponential function solutions, under the choice of the suitable values of parameters, the 3D, 2D and the contour graphs are plotted. The perspective view of the complex exponential function solutions can be seen in the 3D graphs of Figs. 1–10. The propagation pattern of the wave along the $x$-axis are illustrated in the 2D graphs which are located at the top right corner of Figs. 1–10. The contour graphs is an alternative of the 3D plots.

5. Conclusion

The IBSEFM has been successfully applied to the $(3+1)$-dimensional time fractional mKdV–ZK equation. Some complex exponential function solutions are constructed. The newly reported results may be useful in explaining the physical meaning to some of the nonlinear models arising from the various fields of nonlinear sciences. The IBSEFM is a powerful tool for handling different nonlinear models that could be expressed in mathematical form. To the best of our knowledge the application of the IBSEFM to the $(3+1)$-dimensional time fractional mKdV–ZK is presented for the first time in this study.

References

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